Any answers?

Self Organised Criticality in the third decade after BTW

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Singapore, 26 Feb 2009

Outline

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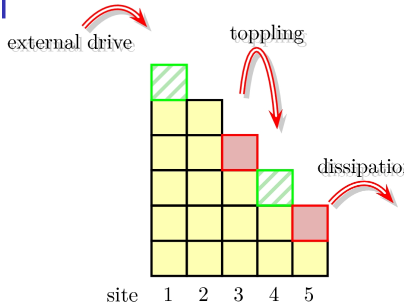
The BTW model



The sandpile model:

- Bak, Tang and Wiesenfeld 1987.
- Simple (randomly driven) cellular automaton $\longrightarrow$ avalanches.
- Intended as an explanation of $1/f$ noise.
- Generates(?) scale invariant event statistics.
- **The physics of fractals.**

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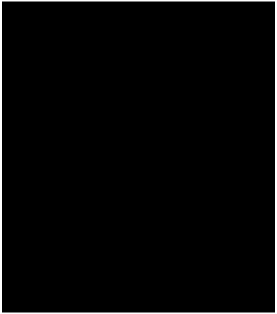

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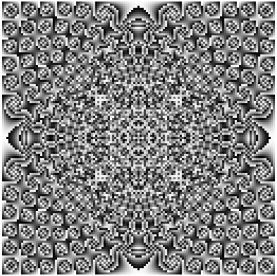
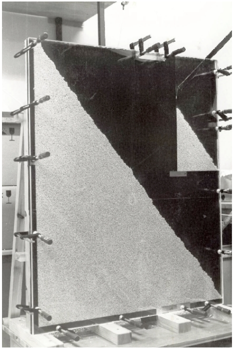


Key ingredients:

- Separation of time scales.
- Interaction.
- Thresholds (non-linearity).
- Observables: Avalanche sizes and durations.

Experiments



Photograph courtesy of V. Frette, K. Christensen, A. Malthe-Sørenssen, J. Feder, T. Jøssang and P. Meakin.

- Large number of experiments and observations:
- Earthquakes suggested by Bak, Tang and Wiesenfeld.
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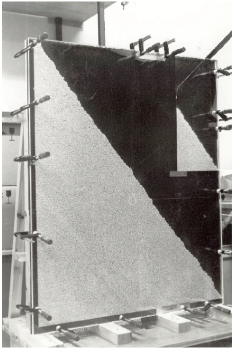

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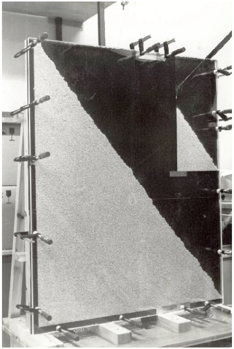

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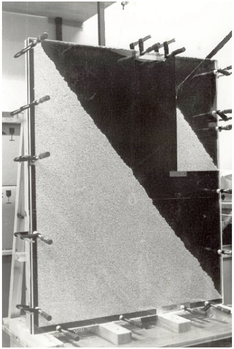

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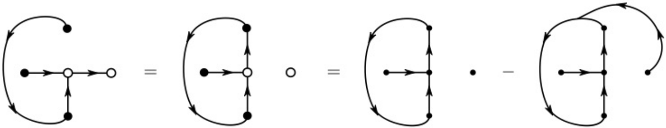
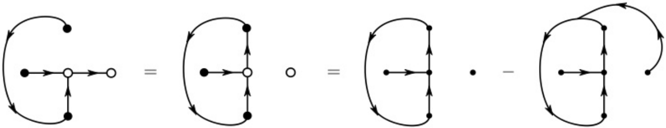
Theory



From Jeng, Piroux, Ruelle (2006)

- Many exact results for the BTW model. No proof of scaling.
  - Most results for Dhar's Abelian version.
  - Mapping to CFT with central charge -2 (spanning trees); $q \to 0$ Potts model
  - Correlation functions known exactly.
  - Wave decomposition.
- Fewer results for other models.
- Other models: Established relation to ordinary critical phenomena.
- No exact solution of non-trivial (long ranged spatio-temporal

Theory



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- Fewer results for other models.
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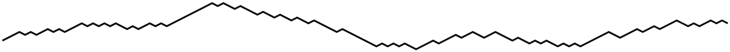
Better Models!

BTW constrained by determinism $\longrightarrow$ stochastic sandpiles!



- **Manna model** has a Langevin equation

$$\partial_t\phi(\mathbf{r},t) = \nu\nabla^2\phi - \mu\phi + \lambda\phi^2 + \omega\rho\phi + \sqrt{2\Gamma^2\rho}\eta(\mathbf{r},t)$$

  and

$$\partial_t\rho(\mathbf{r},t) = \nu_\rho\nabla^2\rho$$

  similar to **directed percolation (C-DP).**
- **Oslo model** implements **quenched Edwards Wilkinson equation** $\longrightarrow$ interfaces!
- Field theories for both still unclear.
- Mechanism of self-organisation still unclear.

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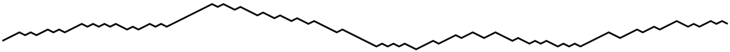

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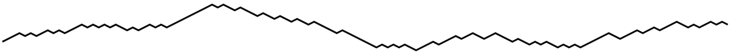

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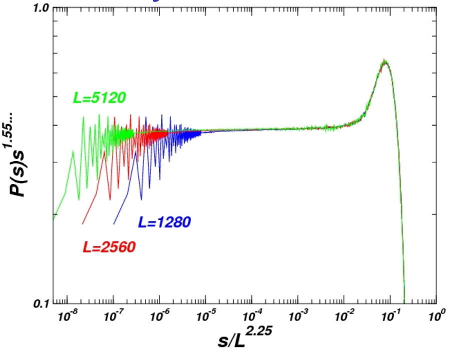
Scaling and Universality



- Only a few models (Manna and Oslo) display solid scaling.
- Robust against (small) changes in the definition $\longrightarrow$ universality.
- Manna and Oslo (apparently) in the same universality class.
- **Is this the only (proper) universality class in SOC?**

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Generic scale invariance

$$\partial_t \phi(\mathbf{r}, t) = (\nu_{\|} \partial_{\|}^2 + \nu_{\perp} \partial_{\perp}^2)\phi + \eta(\mathbf{r}, t)$$

- *Generic* scale invariance (Hwa and Kardar, 1989, and Grinstein, Lee and Sachdev 1990)
- No mass term $-\epsilon\phi$ on the right $\longrightarrow$ conservative dynamics.
- Anisotropy required in the presence of conserved noise.
- Non-trivial exponents in the presence of non-linearities and non-conserved noise.
- Concept abandoned with the arrival of non-conservative models (FFM [1990], OFC [1992], BS [1993]).

The Absorbing State Mechanism

Dickman, Vespignani, Zapperi 1998

- SOC model: activity ρ_a leads to dissipation
- dissipation reduces particle density ζ
- density is reduced until system is inactive
  $\longrightarrow$ absorbing phase
- external drive increases particle density
  $\longrightarrow$ back to active phase

An SOC model can be seen as an AS model that drives itself into the inactive phase by dissipation ϵ and is pushed back into the active phase by external drive h.

$$\dot{\zeta} = h - \epsilon\rho_a \xrightarrow{\text{stationarity}} \rho_a = h/\epsilon$$

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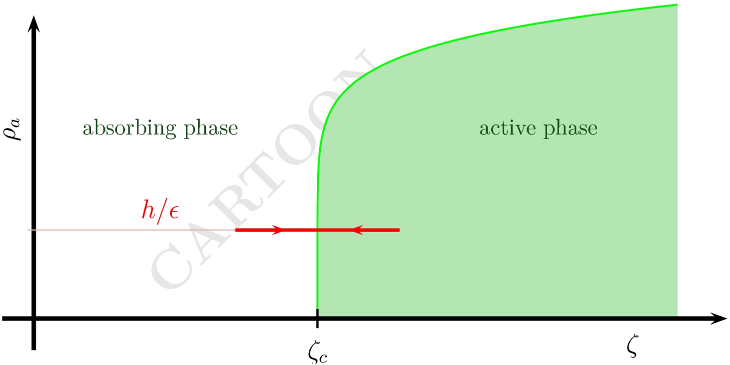

Idea: SOC drives $h/\epsilon = \rho_a$ to 0 as $L \to \infty$

Leading orders: $h(L) = h_0 L^{-\omega}$ and $\epsilon(L) = \epsilon_0 L^{-\kappa}$

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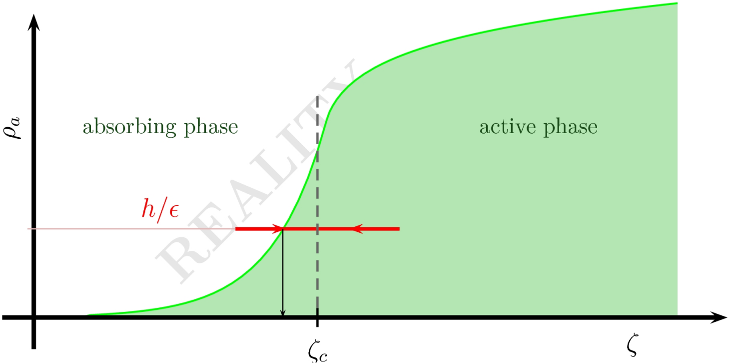

Problem: SOC exponents would be affected by the way how driving and dissipation are implemented $\longrightarrow$ no universality.

Fey, Levine and Wilson suggest that critical point is not reached.

Any Answers?

- Does SOC exist in computer models? Yes. Manna and Oslo models are robust and universal.
- Does SOC exist in nature or experiments? Possibly, superconductors and granular media.
- Is SOC ubiquitous? Apparently not.
- Is SOC understood? Maybe, AS Mechanism suggested, but has problems.
- Is it worth understanding? Certainly: Understanding of long-range correlations in nature and criticality without tuning.

Thanks!

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