From Classical Critical Phenomena to Species Borders

An Overview

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Imperial College London

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Outline

Outline

Classical Critical Phenomena

A Historical Overview I

- Andrews (1869): The critical point
- Van der Waals (1873): Equation of state
- Weiss (1907): Ferromagnetism
- Lenz (1920): Ising model
- Ising (1925): Solution of 1D-Ising
- Ehrenfest (1933): Classification of transitions
- Landau (1937): Unified theory, universality
- Kramers and Wannier (1941): T_c for 2D-Ising
- Onsager (1942): Solution of 2D-Ising
- Yang (1952): $M(T)$ for 2D-Ising

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Classical Critical Phenomena

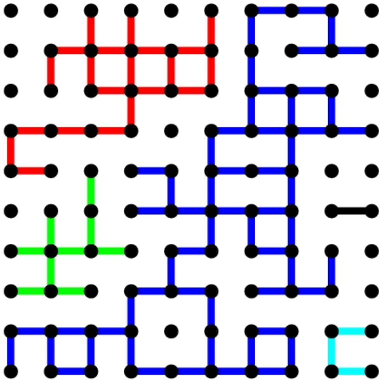
A Historical Overview II

- Stueckelberg and Petermann (1953): Ultraviolet renormalisation
- Gell-Mann and Low (1954): Renormalisation scheme in QED
- Domb and Hunter, and Widom (1965): Scaling hypothesis
- Kadanoff (1966): Generalised scaling and block spins
- Wilson (1971): Use RG in critical phenomena
- Wilson and Fisher (1972): Small parameter $\epsilon = d_c - d$
- Gross, Politzer and Wilczek (1973): Asymptotic freedom

Outline

Classic Example: Bond Percolation

The Model



- On a 2D grid, bonds are active with probability p
- Cluster: set of sites connected through active bonds
- Temperature-like variable: p (drives transition)

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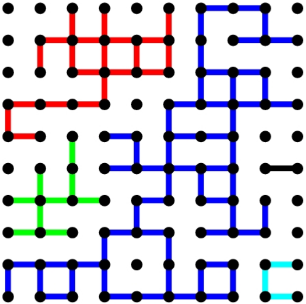

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- Introduce site-site correlation function $g(\vec{r}_1, \vec{r}_2)$
- Asymptotically: Exponential decay $g(\vec{r}_1, \vec{r}_2) \propto \exp(-|\vec{r}_1 - \vec{r}_2|/\xi)$
- At $p = p_c$ the correlation length ξ diverges
  $\rightarrow$ **No characteristic scale**

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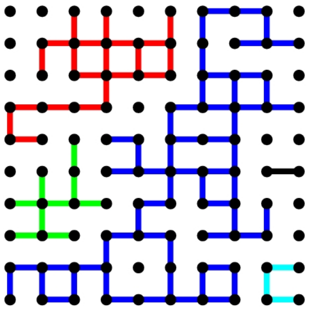

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- History:
  - Three dimensional polymers: Flory 1941
  - Mathematics: Hammersley and Broadbent 1954
    $p_c = 1/2$ conjectured in 1955, proven (Kesten) 1980
  - Renaissance: CFT 1992, SLE 2001

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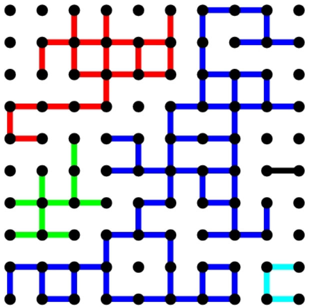

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- Observables...
  - Order parameter θ, percolation probability (coverage by infinite cluster): vanishes in one phase, picks up in the other
  - Cluster size distribution (cluster number density) $\mathcal{P}(s)$
  - Crossing probability E_p (cluster connects two opposite sides)

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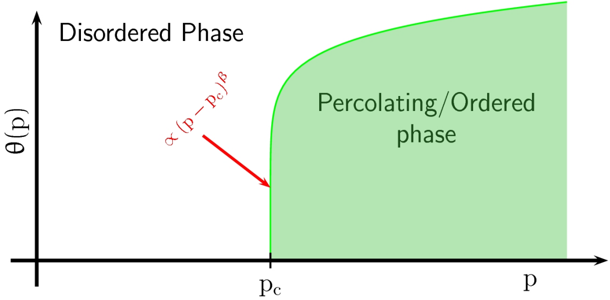

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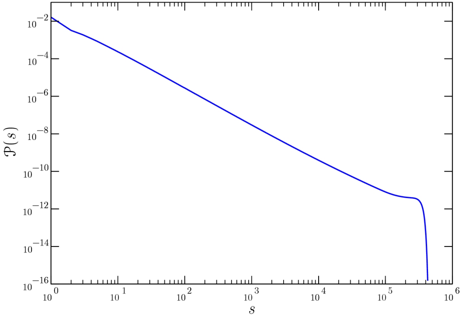

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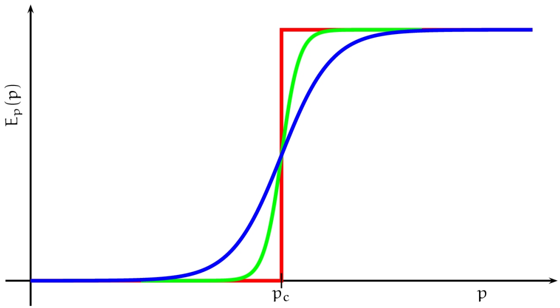

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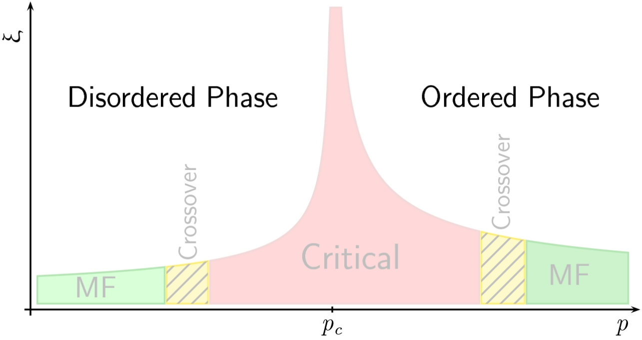
Scanning through the transition



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- Crossover region: Fluctuations start to take over
- Critical region: Non-trivial scaling
- Crossover towards ordered phase
- Warning: Gaussian Theory (trivial theory) does not order

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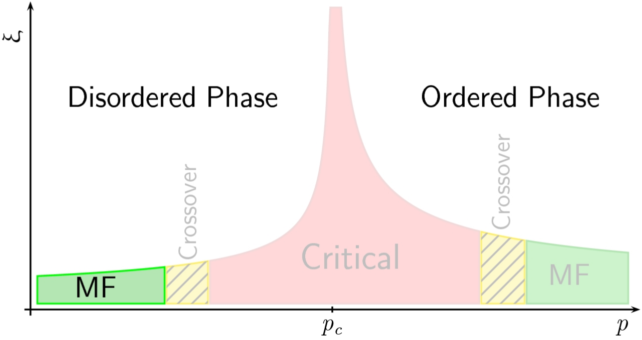

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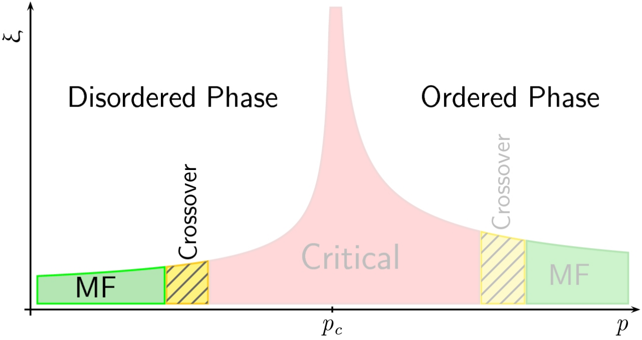

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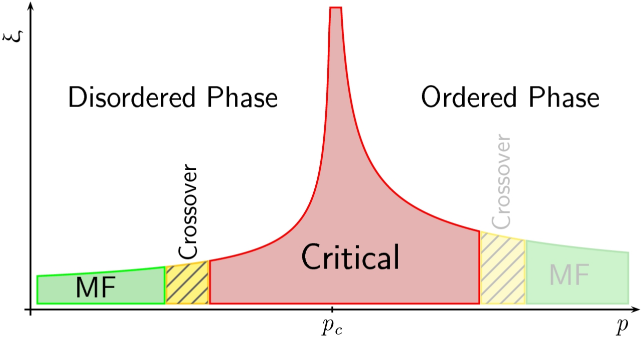

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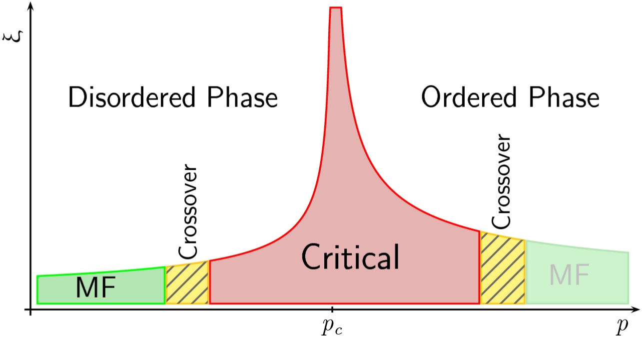

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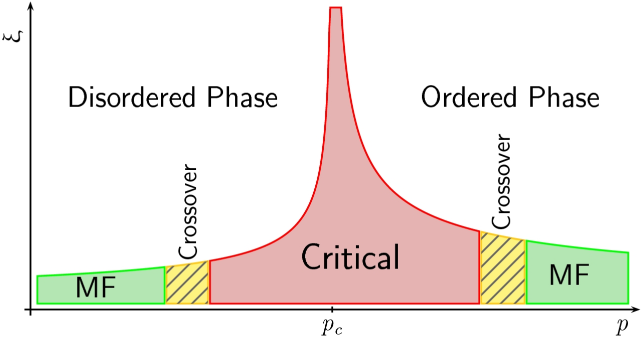

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Scaling around the critical point ($p = p_c$)

The infinite system

Scaling

- Divergent correlation length: $\xi \propto |p - p_c|^{-\nu}$
- Observables and parameters are related by scaling relations
  Example: $\langle s \rangle \propto |p - p_c|^{-(2-\tau)/\sigma}$
- Observables "look the same under rescaling": ...
- Even in infinite systems: Scaling is asymptotic (lower cutoff)
- Infinite system away from critical point:

$$\mathcal{P}(s) = a s^{-\tau} \mathcal{G}\left(s/\xi^D\right)$$

Universal scaling function: $\mathcal{G}$

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  - Probability for cluster size s is $\mathcal{P}(s)$
  - Probability for cluster size $s' = 2s$ is a multiple of $\mathcal{P}(s)$
  - This multiple is the same, independent of s
  - $\mathcal{P}(s)$ does not possess an **intrinsic scale**
  - $\mathcal{P}(s)$ has the form of a power law: $\mathcal{P}(s) = as^{-\tau}$
- Even in infinite systems: Scaling is asymptotic (lower cutoff)
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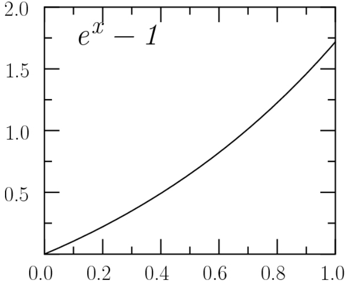
What is so striking about scaling?

Part I: Self-similarity!

Central idea: "Power law" means no intrinsic scale



The function $e^x - 1$ on a small scale

The function $e^x - 1$ on a large scale

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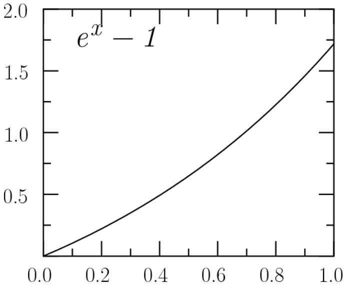

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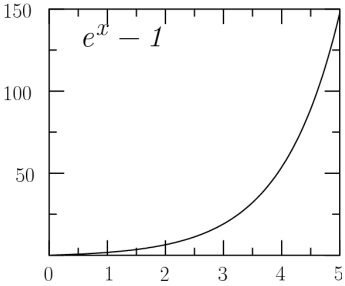

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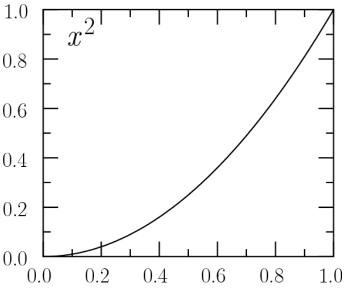

The function x^2 on a small scale

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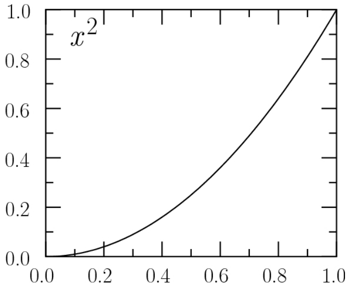

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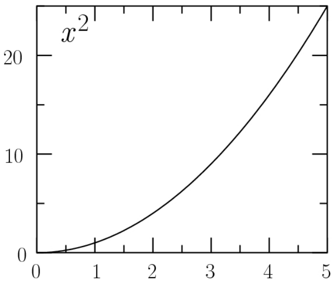

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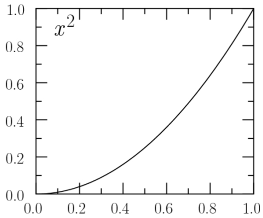
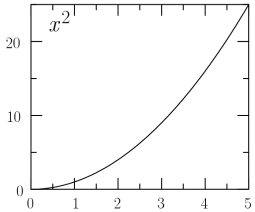
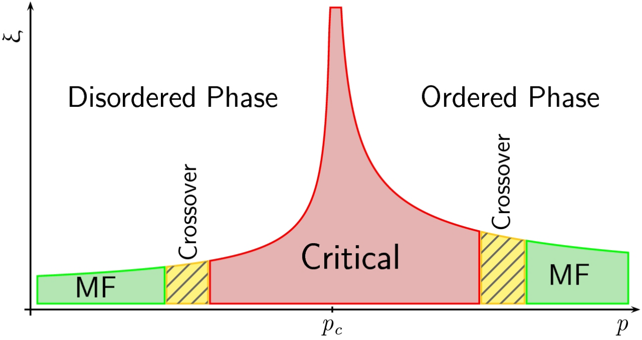




- Trivial powerlaws (can) correspond to dimensional constraints
- Even trivial powerlaws (can) have deep physical meaning
- Non-trivial powerlaws
  - Need finite (microscopic) scale for dimensional consistency
  - Are *not* result of dimensional constraints

Scaling in finite systems



- Infinite systems: Original scenario
- Finite system: Critical scaling where $\xi \ll L$
- Crossover into finite size scaling region
  width: $L^{-1/\nu}$

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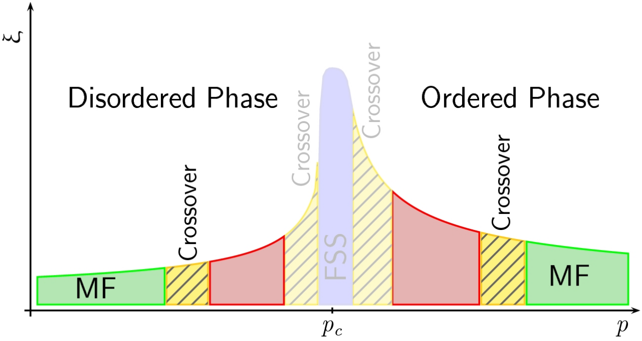

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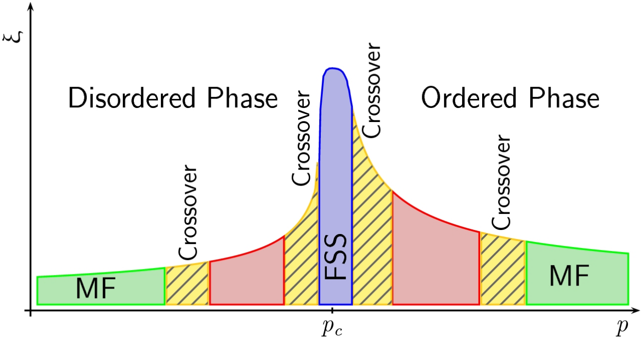

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Part II: Universality

Scale invariance and Universality

- Single scale: correlation length
- Universality classes – universal quantities
  - Exponents
  - Amplitude ratios
  - *Effectively same physics everywhere in asymptotia*
- Many microscopic (interaction) details irrelevant for universal features
- However: Universality of *long range behaviour only*
- Non-universal: p_c, amplitudes, lower cutoff, amplitude of upper cutoff

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Non-Equilibrium Critical Phenomena

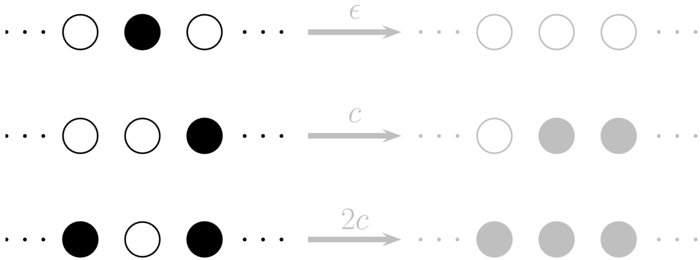
Example: Absorbing State Phase Transition

Absorbing State Phase Transition

- System runs “until it hits an absorbing state”
- Two phases: Eventually in absorbing state or always active
- Continuous transition between absorbing and active
- Problem: Finite systems
- Paradigm: directed percolation
- Grassberger: Single absorbing state? DP!

Absorbing State Phase Transition

Contact Process



- CP *is* DP with asynchronous updates
- Active site become inactive: rate ϵ
- Inactive site become active: rate c per active neighbour
- Rescale time, so that ϵ effectively disappears and $c \to \lambda = c/\epsilon$
- Parameter that drives transition: $\lambda \to \lambda_c = 1.6488\ldots$

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Absorbing State Phase Transition

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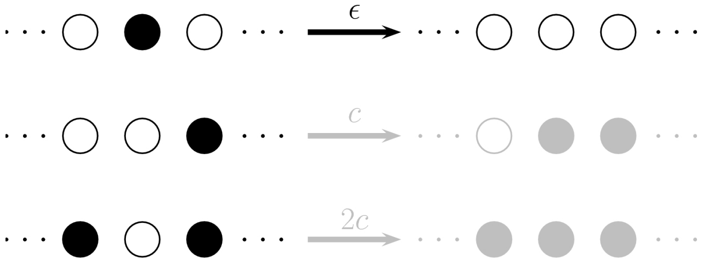

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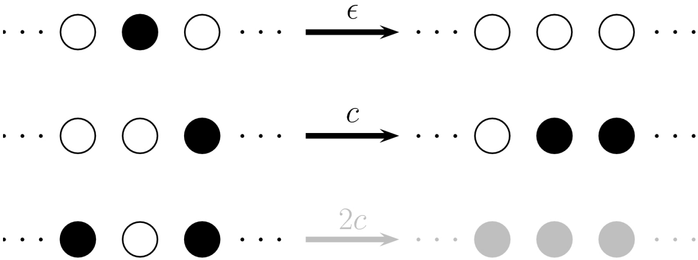

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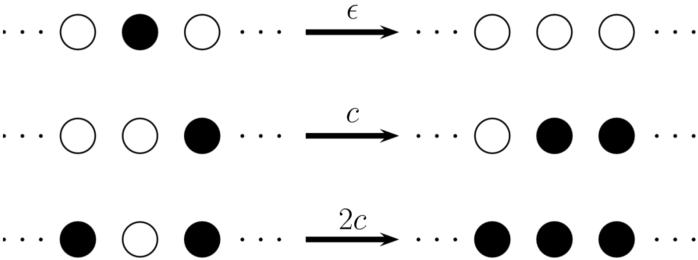

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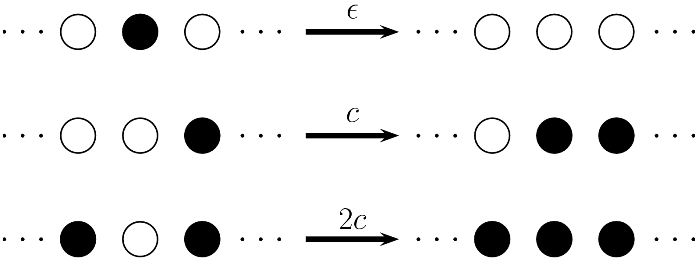

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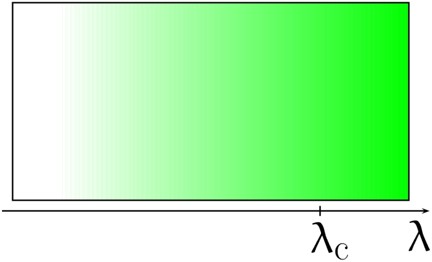
   - Historical Overview
   - Classic Example: Bond Percolation
   - Scaling and Finite Size Scaling
2. Non-Equilibrium Critical Phenomena
   - Absorbing State Phase Transition
   - Species Borders
   - Summary

Contact Process in Population Dynamics

Spatially varying temperature-like variable



- Biological systems: Spatially varying λ
  (like scanning through all λ simultaneously)
  $\lambda = \lambda(x) = \lambda_c + \lambda' x$
- Continuum vs. lattice (vanishing vs. finite ξ)

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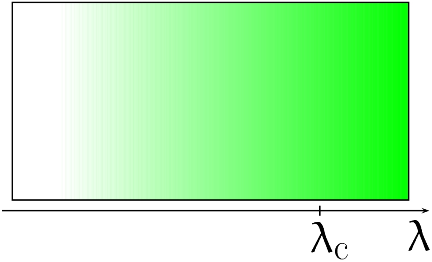

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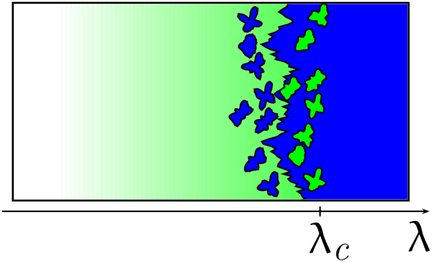
The Contact Process with $\lambda(x)$

Part I: The interface



- Interface extends inside the disordered region
  "Autonomous" fluctuations only in ordered phase
  After long times: Disordered phase fluctuates by invasion only
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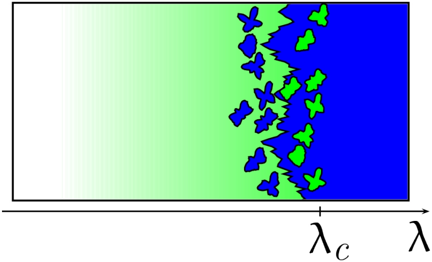
  Properties? Wetting?
- What is the effect of higher order interactions?

The Contact Process with $\lambda(x)$

Part I: The interface



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The Contact Process with $\lambda(x)$

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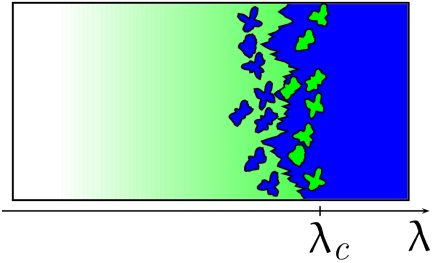

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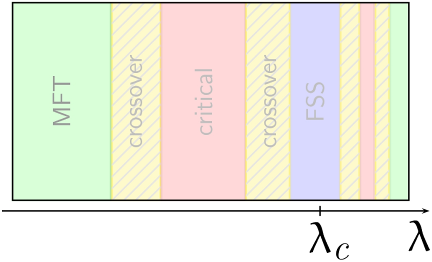
The Contact Process with $\lambda(x)$

Part II: Scaling regions



- Spatial behaviour reflects critical regions of equilibrium system
- Where $L \gg \xi$: critical scaling
- FSS region: $L \approx \xi$...

The Contact Process with $\lambda(x)$

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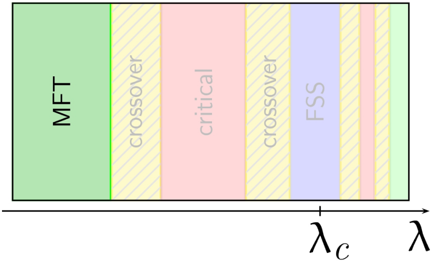

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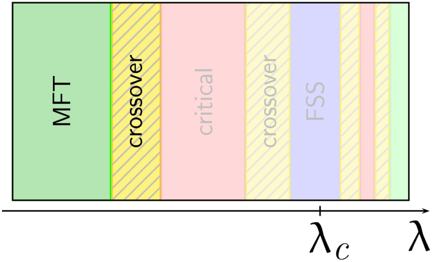

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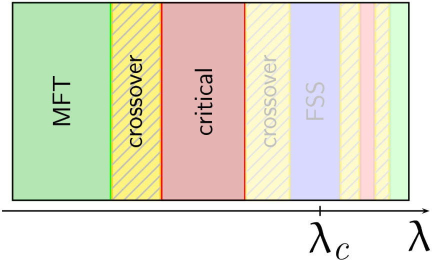

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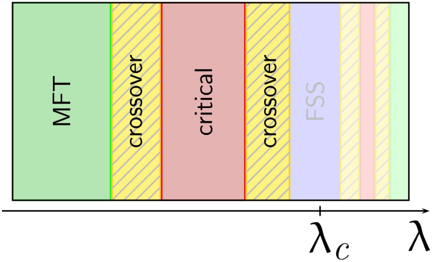

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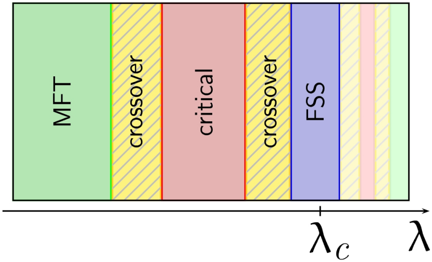

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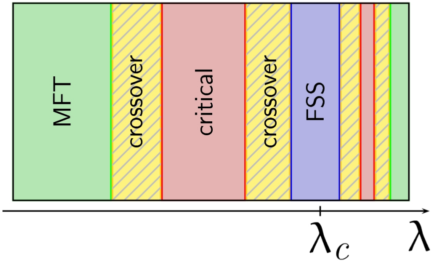

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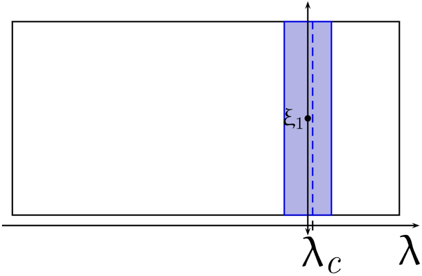
The Contact Process with $\lambda(x)$

Part III: Identifying the FSS region ΔX



What is the size Δx of the scaling region?

- "Naïve" FSS region: $\xi_{\text{bulk}} \approx L$

  $\Delta x \propto \lambda'^{\,-1} L^{-1/\nu}$

  Problem: Width of correlated region underestimated

- Better guess(?): Correlated patches

  Identify a patch around λ_c, within which every point can at least

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The Contact Process with $\lambda(x)$

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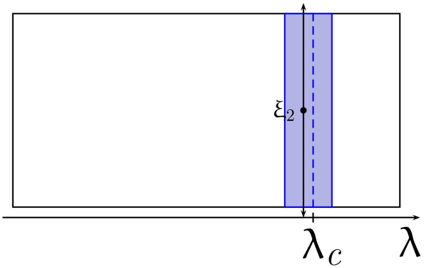

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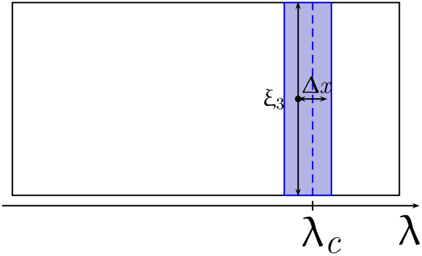

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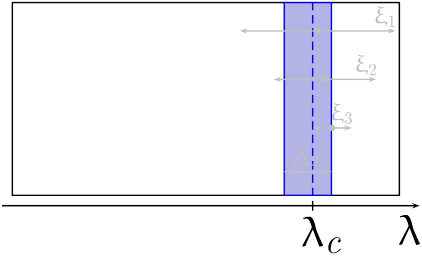

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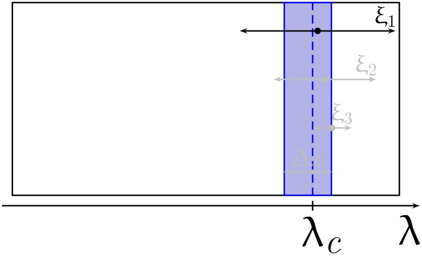

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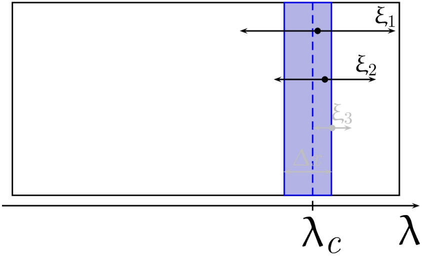

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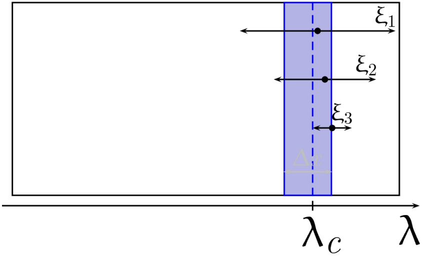

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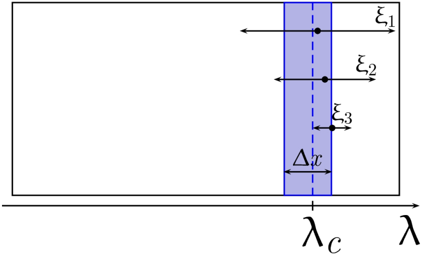

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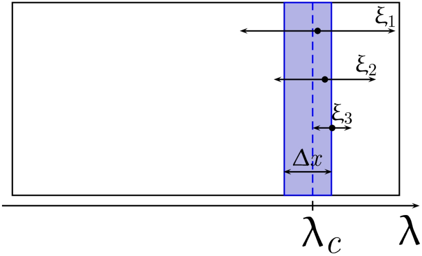

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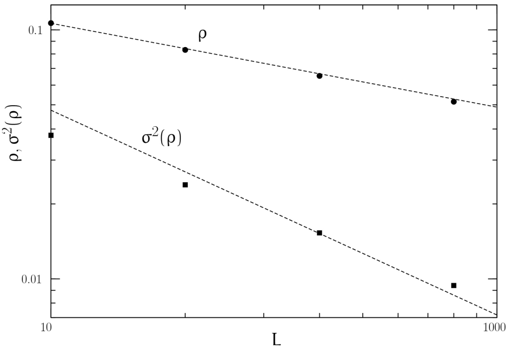
Part IV: Some Numerical Results



Numerics compatible to “scaling region argument”, but not perfect

Outline

Summary

- Phase transition driven by spatial variation of temperature-like variable: Theoretically very appealing
- In different phases, standard methods should apply
- Open problem: What is the size and the scaling of the scaling region?
- Further numerics needed

Many thanks to Nicholas P. Moloney, Zoltán Rácz and Beáta Oborny!