

The universal rotation curve of spiral galaxies

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Accepted 2006 June 12. Received 2006 June 6; in original form 2006 April 24

ABSTRACT

The observed strong dark-to-luminous matter coupling is described by a bias relation between visible and dark matter sources. We discuss the bias which emerges in the case where the topological structure of the Universe at very large distances does not properly match that of the Friedman space. With the use of such ‘topological’ bias, we construct the universal rotation curve (URC) for spirals which happens to be in striking agreement with the empirically known URC. We also show that the topological bias explains the origin of the Tully–Fisher relation ($L \sim V^4$) and predicts peculiar oscillations in the URC with a characteristic length $\sim \sqrt{L}$.

Key words: galaxies: kinematics and dynamics – galaxies: spiral – dark matter.

1 INTRODUCTION

It has long been known (Persic, Salucci & Stel 1996, hereafter PSS) that the shape of the rotation curves of spirals is rigidly determined by a single global parameter, e.g. luminosity or the number of baryons in a galaxy. This feature was stressed in PSS by an empirical construction of a universal rotation curve (URC) which describes quite well the rotation velocity at any radius and for any galaxy as a function of, say, the galaxy luminosity only. It follows that the distribution of the dark matter (DM) in galaxies carries a universal character as well, and is a function of the luminous mass. Note that the standard cold DM (CDM) models fail to explain this strong dark-to-luminous matter coupling, for an obvious reason: in any model where DM is built from hypothetical non-baryonic particles (e.g. CDM, worm DM, or self-interacting DM) the number of the DM particles in the halo of a galaxy is, essentially, a free parameter, and relating it to the number of baryons in the galaxy requires some very strong non-linearity. Moreover, it is well established that the DM density in galaxies shows an inner core, i.e. a central constant density region (e.g. see Gentile et al. 2004 and references therein; Wdruke, de Blok & Walter 2003; de Blok & Bosma 2002; for spirals and Gerhard et al. 2001; Borriello, Salucci & Danese 2003 for ellipticals), which is in clear conflict with the predictions of Λ CDM models yielding Navarro–Frenk–White (NFW) type profiles with a cusp (e.g. $\rho_{\text{DM}} \sim 1/r$) in the central region of a galaxy (Navarro, Frenk & White 1996).

The strong coupling between DM haloes and baryons (see also Donato, Gentile & Salucci 2004) definitely requires some new physics. The coupling can be described by a rigid relation between the sources of dark, ρ_{DM} , and visible, ρ_{L} , matter; the so-called bias relation (Kirillov 2006). In the linear case the most general form of

the bias relation is

$$\rho_{\text{DM}}(x) = \int b(x, x') \rho_{\text{L}}(x') dV'. \quad (1)$$

The homogeneity assumption $b(x, x') = b(x - x')$ allows one to fix empirically the bias operator b_{emp} . Indeed, in this case the Fourier transform of the bias relation (1) gives

$$\rho_{\text{DM}}(t, k) = b(t, k) \rho_{\text{L}}(t, k) \quad (2)$$

where we added a dependence on time to account for the cosmic evolution. The empirical bias function $b_{\text{emp}} = \rho_{\text{DM}}(t, k)/\rho_{\text{L}}(t, k)$, in virtue merely of its definition, will perfectly describe DM effects at very large scales (i.e. in the region of linear perturbations).

The present Universe is not quite homogeneous though, e.g. it is not uniform at galaxy scales. Still, we would expect relation (2) to hold in the geometrical optics limit (i.e. for rather short wavelengths as compared to the Hubble scale, or to a cluster scale when a single galaxy is considered). Parameters of the bias function may then vary for different spatial regions, i.e. b_{emp} may include an additional slow dependence on the location in space: $b = b_{\text{emp}}(t, k, x)$. In order to fit observations, any theoretical source of DM should reproduce properties of the bias function b_{emp} in detail.

In linear gravity, the bias relation (2) can be interpreted as a modification of the Newton law:

$$\frac{1}{r^2} \rightarrow \frac{2}{\pi} \int_0^\infty [1 + b(k)] \frac{\sin(kr) - kr \cos(kr)}{kr^2} dk. \quad (3)$$

The asymptotically flat rotation curves in galaxies require that the correction to the Newton’s potential should be logarithmic, i.e. the gravitational acceleration should switch from r^{-2} to r^{-1} . This, according to (3), implies $b(k) \sim k^{-1}$, or

$$b(x - x') \sim |x - x'|^{-2} \quad (4)$$

at galaxy scales. In fact, observations suggest the same behaviour of $b(x - x')$ for much larger scales (Kirillov 2006). Indeed, the distribution of the luminous mass shows characteristically fractal

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behaviour: the mass $M_L(r)$ within the ball of radius r grows, essentially, as $\sim r^D$ with $D \simeq 2$ on distances up to at least 200 Mpc (Pietronero 1987; Ruffini, Song & Taraglio 1988; Labini, Montuori & Pietronero 1998). The bias function (4) leads then to $M_{\text{DM}}(r) \sim r^3$, i.e. to a homogeneous distribution of the DM and, hence, of the total mass. Thus, bias (4) is just one that reconciles the two seemingly contradictory observational facts: the fractal distribution of baryons with the dimension $D \simeq 2$ and the large-scale homogeneity of the metric.

A theoretical scheme capable of explaining the origin of such bias was proposed in (Kirillov 1999; Kirillov & Turaev 2002). It was shown there that processes involving topology changes during the quantum stage of the evolution of the Universe unavoidably (and, in fact, model-independently) lead to a scale-dependent renormalization of the constant of gravity, G , and this effect can be imitated by the emergence of DM, the distribution of which is linearly related to the distribution of actual matter. Importantly, assuming the thermal equilibrium during the quantum stage predicts in a unique way a very specific form of the bias function (Kirillov & Turaev 2002; Kirillov 2003)

$$b(k) = \frac{\mu}{\sqrt{\kappa^2 + k^2}} \quad \text{for } k < \mu. \quad (5)$$

where $\mu \sim T_{\text{pl}} a(t_{\text{pl}})/a(t)$ has the meaning of the primordial temperature at which the topology has been tempered and $\kappa \sim ma(t_{\text{pl}})/a(t)$ is the mass of primordial scalar particles if they exist. This means that at scales $k \lesssim \kappa$ the bias becomes constant: $b(k) = \mu/\kappa$ and the standard Newton's law is restored. However, at galaxy scales there should be $k \sim \mu \gg \kappa$, so in what follows we take $\kappa = 0$ in (5).¹ Thus in the coordinate representation the bias takes the form

$$\begin{aligned} b(\mathbf{r}, t) &= \frac{1}{2\pi^2} \int_0^\mu [b(k)k^3] \frac{\sin(kr)}{kr} \frac{dk}{k} \\ &= \frac{\mu}{2\pi^2 r^2} [1 - \cos(\mu r)]. \end{aligned} \quad (6)$$

Bias (5) is of the form of (4), so it predicts the logarithmic correction to the Newton's potential for a point source: $\delta\phi \sim (1/R_0) \ln r$ at $r > R_0$, where $R_0 = \pi/(2\mu)$ (see for details Kirillov & Turaev 2002; Kirillov 2006). Thus, the parameter R_0 plays the role of the scale at which DM starts to show up, so in galaxies it has to be estimated as a few kpc. As bias (5) has a thermodynamical origin, there have to be certain fluctuations in the value of R_0 (this effect is analyzed in the next section).

In the present Letter we demonstrate that bias (5), (6) gives very good agreement with the empirical URC constructed in PSS and, therefore, it seems to be reasonable to believe that the non-trivial topological structure of the Universe indeed gives a correct explanation of the DM phenomenon.

2 THE TULLY–FISHER RELATION

The parameter μ in (6) represents the temperature at which the topology has been tempered (Kirillov & Turaev 2002; Kirillov 2003). We expect that in the very early Universe it had the order of the Planck temperature T_{pl} (recall that at $T > T_{\text{pl}}$ quantum gravity effects are thought to dominate). When, on the further stages of the Universe evolution, topology changes were suppressed, this parameter evolved as $\mu \sim T_{\text{pl}} a(t_{\text{pl}})/a(t)$, where a is the scale factor. Note,

¹For example, inflationary scenarios suggest $m \sim 10^{-5} m_{\text{pl}}$ and, therefore, $\mu/\kappa \sim 10^5$. However, so far scalar particles have not been observed which makes us think that scalar fields are no more than phenomenological objects.

however, that the present value of μ has the sense of the primordial temperature. As it is extremely small: $\mu \sim 10^{-23} T_\gamma$ where T_γ is the temperature of CMB radiation ($T_\gamma \simeq 2.7$ K), we have to admit the existence of a specific phase in the past when the non-trivial topological structure might decay (Kirillov & Turaev 2002; Kirillov 2003), causing a certain re-heating of matter. During the decay phase, μa was a decreasing function of time, i.e. the scale $R_0 = \pi/(2\mu)$, that corresponds to the cross-over from the standard Newton's law to the logarithmic behaviour of the potential of a point mass, grew faster than the scale factor $a(t)$.

Note that the homogeneity of the Universe requires the total mass distribution (luminous plus dark components) to have a constant density in space. With the bias of form (5), (6), this corresponds to a fractal distribution of baryons (Kirillov & Turaev 2002; Kirillov 2003), i.e. the number of baryons within the sphere of a radius $R > R_0$ behaves as

$$N_b(R) \simeq \nu R^D \quad (7)$$

with $D \simeq 2$, while for $R < R_0$ the fractal distribution is unstable (for the Newton's law restores and baryons dominate over the DM). The increase of R_0/a allows one to assume that in the very early Universe there was a moment t_* when $N_b[R_0(t_*)] < 1$, i.e. baryons had the fractal distribution (7) at all scales. After the topology decay phase, as the scale $R_0(t)$ 'jumps' towards a new, higher value, the fractal distribution is preserved at scales larger than R_0 , but it becomes unstable on smaller scales. The instability develops and baryons under a certain scale of order R_0 start to redistribute, governed by Newtonian dynamics. This means that we can relate R_0 to the scale of galaxy formation. Then, according to (7), we should expect the number of baryons in a galaxy to be

$$N_b \simeq \nu' R_0^D, \quad (8)$$

with the values of R_0 and ν' corresponding to the moment when a galaxy started to form. Note, however, that during the formation of a galaxy the value of R_0 switches off from the Hubble expansion (Kirillov 2006), i.e. law (8) remains valid for the present-time values of N_b and R_0 .

It is easy to see that this relation leads directly to the Tully–Fisher law $L \sim V^4$ (Tully & Fisher 1977), where L is the luminosity and V is the rotation velocity of a galaxy. Indeed, recall that R_0 fluctuates in space: small spatial fluctuations of the primordial temperature μ ($\Delta\mu/\mu \sim \Delta T_\gamma/T_\gamma$) represent seeds for the present-time scatter in the local value of $\mu = \pi/(2R_0)$ in different galaxies. Accordingly, the masses of galaxies $M \sim m_b N_b$ (where m_b is the baryon mass) fluctuate as

$$M \simeq m_b \nu' R_0^D. \quad (9)$$

This fixes the choice of

$$\mu = \mu_g \simeq \frac{\pi}{2} \left(\frac{M_g}{m_b \nu'} \right)^{-1/D} \quad (10)$$

in the bias (5), (6) for any given galaxy. Sufficiently far from the centre, the galaxy can be considered as a point-like object, so (3), (5) yield the following law for the gravitational acceleration at a sufficient distance from the edge of the optical disc:

$$g = \frac{GM_g}{r^2} \left\{ 1 + \frac{2}{\pi} [\mu_g r - \sin(\mu_g r)] \right\}. \quad (11)$$

As $V_\infty^2/r = g(r)$, the Tully–Fisher relation for the asymptotic rotation velocity V_∞ follows now from (11) and (10):

$$V_\infty^2 \simeq \frac{2}{\pi} GM_g \mu_g \implies L_g \sim M_g \simeq \left(\frac{V_\infty}{a} \right)^\beta, \quad (12)$$

with $\beta = 2D/(D - 1)$ and $a^2 = (2/\pi)G(m_b v')^{1/D}$. Thus, in our interpretation of the DM, the Tully–Fisher law reduces to relation (10) which, in turn, can be read as an indication of the fractality of the primordial distribution of baryons with the dimension $D \simeq 2$ (see equation 7).

3 ROTATION CURVE OF SPIRALS

Let us now compute the rotation curve (RC) of a galaxy modelled by an infinitely thin disc with surface mass density distribution $\rho_L = \sigma e^{-r/R_D} \delta(z)$. From (1), (6), we find for the DM halo density (we use the notations $x = r/R_D$ and $\lambda = \mu R_D$)

$$\rho_H(\mathbf{x}) = \frac{\lambda \sigma}{2\pi^2 R_D} \int \frac{e^{-y}}{|x-y|^2} (1 - \cos(\lambda|x-y|)) d^2y \quad (13)$$

where y lies on the plane $z = 0$, while \mathbf{x} is the three-dimensional vector. For the sake of convenience, we present the Fourier transform

$$\rho_H(k, k_z) = \frac{\mu}{\sqrt{k^2 + k_z^2}} \frac{M_L}{[(kR_D)^2 + 1]^{3/2}} \theta\left(\mu - \sqrt{k^2 + k_z^2}\right) \quad (14)$$

where θ is the step function: $\theta(u) = 0$ for $u < 0$, $\theta(u) = 1$ for $u > 0$, and $M_L = 2\pi \sigma R_D^2$ is the (non-dark) mass of the galaxy.

First of all we note that this distribution is quite consistent with the observed cored distribution (Gentile et al. 2004). Indeed, in the central region of the galaxy

$$\rho_H(0) = \frac{M_L}{(2\pi)^2 R_D^2} \lambda \ln(1 + \lambda^2), \quad (15)$$

while for $x \gg 1$ we find

$$\rho_H(x) \approx \frac{2\rho_H(0)}{\ln(1 + \lambda^2)} \frac{[1 - \cos(\lambda x)]}{x^2}. \quad (16)$$

If we neglect the oscillating term and compare this with the pseudo-isothermal halo $\rho = \rho_0 \alpha^2 / (\alpha^2 + x^2)$ we find for the core radius

$$\alpha^2 = \frac{R_C^2}{R_D^2} = \frac{2}{\ln(1 + \lambda^2)}. \quad (17)$$

According to PSS, the core radius can be estimated as

$$\alpha = 4.8 (L/L_*)^{1/5} \quad (18)$$

with $\log L_* = 10.4$, which makes λ a certain function of the luminosity.

Consider now circular velocities predicted by the above mass distributions. For the disc contribution to the equilibrium circular velocity, we get (PSS)

$$\begin{aligned} \frac{V_D^2}{V_\infty^2} &= f_D(x, \lambda) \\ &= \frac{\pi x^2}{4 \lambda} \left[I_0\left(\frac{x}{2}\right) K_0\left(\frac{x}{2}\right) - I_1\left(\frac{x}{2}\right) K_1\left(\frac{x}{2}\right) \right], \end{aligned} \quad (19)$$

and for the dark halo contribution we find from (14) the expression

$$\frac{V_H^2}{V_\infty^2} = f_H(x, \lambda) = \frac{x}{\lambda} \int_0^\lambda \frac{\sqrt{\lambda^2 - k^2}}{[\sqrt{(k^2 + 1)}]^3} J_1(kx) dk, \quad (20)$$

where J_n, I_n, K_n are the Bessel and the modified Bessel functions and $V_\infty^2 = (GM_g/R_D)(2/\pi)\lambda$. Thus for the rotation curve we find the expression

$$V^2(x, \lambda) = V_\infty^2 [f_D(x, \lambda) + f_H(x, \lambda)]. \quad (21)$$

As we see, the shape of the rotation curve indeed depends on one parameter; λ . Via relation (9), or equivalently (12), λ is expressed

as a function of the total number of baryons N_b in the galaxy; there is, however, an uncertainty in λ due to the variation of the ratio M/L for different galaxies. At the moment of the galaxy formation $R_D \sim R_0$, which corresponds to the same initial value, $\lambda \sim 1$, in all galaxies. At subsequent stages of the evolution, λ becomes different in different objects. Indeed, in smaller galaxies supernovae are more efficient in removing the gas from the central (star-forming) region of a galaxy than in bigger galaxies (e.g. see Shankar et al. 2006, and references therein) and this means that in smaller objects the disc has a smaller baryonic density (a lower surface brightness) and the ratio $\lambda \simeq R_D/R_0 \gg 1$.

To compare expression (21) with that from PSS we rewrite it as

$$\frac{V^2(x, \lambda)}{V_{\text{opt}}^2} = \frac{f_D(x, \lambda) + f_H(x, \lambda)}{f_D(3.2, \lambda) + f_H(3.2, \lambda)} \quad (22)$$

where $x = 3.2$ corresponds to the optical radius of a galaxy. The plot of this curve for different values of λ is presented in Fig. 1.

While the similarity of our RC (22) with the empirical URC of PSS is quite good, we note that the topological bias (6) predicts a new feature in RCs—specific oscillations in the DM density with the characteristic wavelength $\ell = 2\pi/\lambda$ (or in dimensional units $\ell \sim M_g^{1/D}$). Indeed at a sufficient distance from the edge of the optical disc (i.e. as $x \gg 3.2$) a galaxy can be considered as a point-like object. Then from (11) for the rotation velocity we find the expression

$$\frac{V^2(x)}{V_\infty^2} = \frac{\pi}{2\lambda x} + 1 - \frac{\sin(\lambda x)}{\lambda x} \quad (23)$$

which shows the presence of a specific oscillations with the decaying amplitude $1/(\lambda x)$. Such oscillations are, in turn, rather difficult (though possible) to extract from observations. Indeed in the case of HSB (high surface brightness) galaxies when $\lambda \lesssim 1$ (i.e. for rather long periods $\ell \gtrsim 2\pi$) the expression (23) gives a very good quantitative approximation to the exact formula (22) starting already from $x = x_{\text{opt}} = 3.2$. However, the reliable RC data available do not usually extend to more than to $x = (2 - 3)x_{\text{opt}}$. In this range oscillations are not established yet and the beginning of oscillations is seen (e.g. see Fig. 1) as RC slopes that are not flat. The slopes observed are known to take values between 0.2 and -0.2 (e.g. see PSS). In the case of low surface brightness (LSB) galaxies, $\lambda \gg 1$ ($\ell \ll 2\pi$), the amplitude of oscillations is somewhat suppressed, $\sim 1/\lambda$, and the small amount of the stellar mass in the range $x_{\text{opt}} < x < 3x_{\text{opt}}$ considerably smooths the oscillations which results in a certain deviation of (23) from the exact expression (22). Moreover, in deriving (22) we do not take into account the presence of gas which due to supernovae does not trace the brightness, i.e. it deviates the exponential profile. Essentially this is true for LSB galaxies. Thus, to observe such oscillations we have either to measure velocities for sufficiently large distances $\sim 10x_{\text{opt}}$ (e.g. for high surface brightness galaxies), or to improve the accuracy of available observational data in LSB galaxies.

4 DISCUSSION AND CONCLUSIONS

As it can be seen from Fig. 1, the topological bias (5) predicted in (Kirillov & Turaev 2002; Kirillov 2003) shows quite a good agreement with observations. Indeed, it repeats all features of the empirical URC of PSS: the amount of DM progressively increases with decreasing luminosity (cf. PSS), DM shows the cored distribution (cf. Gentile et al. 2004) with the strong correlation (17) between the core radius and the disc size (cf. Donato et al. 2004), and the Tully–Fisher relation (Tully & Fisher 1977) is explicitly present

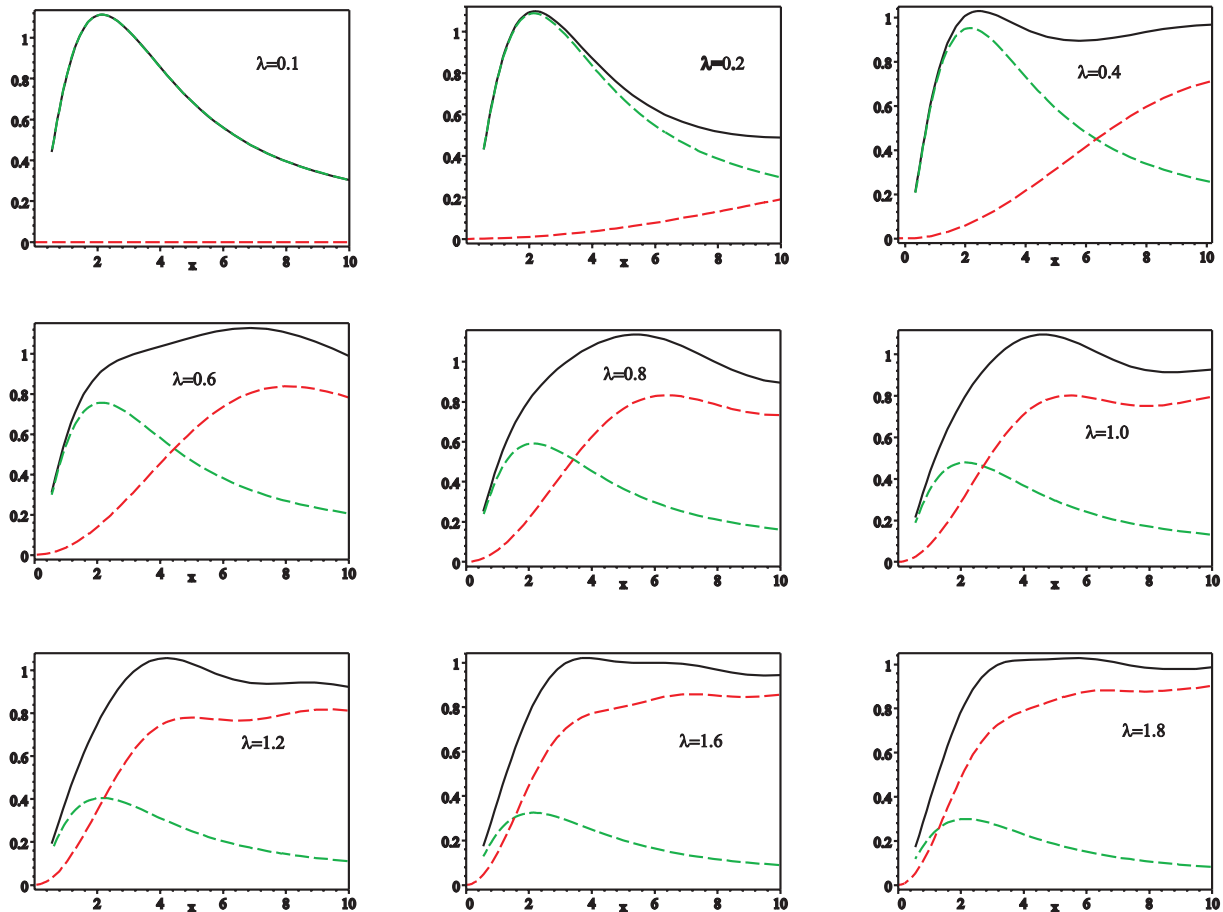


Figure 1. The rotation curves $V^2(x)/V_{\text{opt}}^2$ vs $x = r/R_D$ for different values of λ . The green and red dashed lines give the visible and DM contributions, respectively, while the black line gives the sum. We see that with the decrease of the luminosity (increase of λ) DM fraction increases in agreement with PSS.

(see 12). There is no doubt that the tuning of a single free parameter M/L allows to fit any RC. At least, this is claimed in e.g. Milgrom & Sanders (2005, and references therein) for the RCs obtained via the Milgrom algorithm of modified Newtonian dynamics (MOND Milgrom 1983), and our RCs are phenomenologically quite close to those, although the physics in our approach is completely different. In this respect we can claim that the topological bias gives a rigorous basis for applying the MOND-type algorithm in galaxies (which, however, allows the Tully–Fisher relation to have $\beta \neq 4$). Therefore, there is enough evidence that bias (1)–(6) gives an adequate description of galaxies.

We repeat that our approach produces as good a fit to the observed RCs as the Milgrom algorithm, known to be quite successful empirically (Milgrom & Sanders 2005, see however Gentile et al. 2004; Donato et al. 2004), can. However, contrary to MOND, our theory remains linear in weak fields, and the superposition of forces holds. In fact, our approach does not presume any modification of the theory and basic equations: there is actually no modification of gravity, while the bias appears merely as a result of a disagreement between the actual topology of the physical space and that of the flat space (e.g. see section 2 in Kirillov 2006). Thus, there is every reason to believe that the DM phenomenon indeed has a topological origin.

Once we accept the bias (1)–(6), the Tully–Fisher relation

$$L \sim V_{\infty}^{\beta} \quad (24)$$

with $\beta = 2D/(D - 1) \simeq 4$ serves as a strong indication of the fractal behaviour in the primordial distribution of baryons with the dimension $D \simeq 2$. Such fractal distribution changes essentially the estimate for the baryon number density in the Universe (e.g. see Kirillov 2006). The currently accepted post-*WMAP* cosmology has (roughly): $\Omega_{\text{total}} = 1$, $\Omega_{\Lambda} \sim 0.7$, $\Omega_{\text{DM}} \sim 0.25$, and $\Omega_{\text{b}} \sim 0.05$, which implies $\Omega_{\text{DM}}/\Omega_{\text{b}} \sim 5$. We stress that such estimates are model-dependent, for they are strongly based on the standard model (e.g. the content, evolution, the homogeneity of the baryon distribution, the power law of initial spectrum of perturbations, etc.).² Moreover, the direct count of the number of baryons gives $\Omega_{\text{b}} \sim 0.003$ for the whole nearby Universe out to the radius $\sim 300h_{50}^{-1}$ Mpc (e.g. see Persic & Salucci 1992), which means that in the standard cosmological models most of baryons are hidden somewhere.

When the topological bias is accepted such estimates require essential revision. Indeed, according to (5), the topological bias modifies the Newton’s law in the range of scales $\mu > k > \kappa$ where the equilibrium distribution of baryons exhibits fractal behaviour. On

²For linear perturbations there are no doubts that if we take the observed spectrum of $\Delta T/T$ and fix an arbitrary model $a(t)$, we find in a unique way the initial primordial spectrum of $\Delta\rho/\rho$. To fix the model we have to know both the measured $\Delta T/T$ and the initial values of $\Delta\rho/\rho$ which are unknown a priori.

scales $k < \kappa$ the standard Newton's law is restored (and baryons cross over to the homogeneity) but dynamically every particle becomes heavier in $1 + \mu/\kappa$ times, which gives for the effective DM fraction $\Omega_{\text{DM}}/\Omega_{\text{b}} \sim 1 + \mu/\kappa$. The scale $R_0 = \pi/(2\mu)$ is directly measured in galaxies by RCs and is estimated as a few kpc. However, the mean value $\langle R_0 \rangle$ for the homogeneous Universe should be 10^2 times bigger (Kirillov 2006). The maximal scale $1/\kappa$ is the scale where the primordial fractal distribution of baryons crosses over to the homogeneity. This scale is not so easy to measure without a detailed investigation. Indeed, the large-scale structure, e.g. the existence of huge (~ 100 – 200 Mpc) voids with no galaxies inside and thin (~ 1 – 5 Mpc) walls filled with galaxies, fits quite well into the fractal picture and suggests only the lower boundary $1/\kappa > 100$ – 200 Mpc. This gives a DM fraction of $\Omega_{\text{DM}}/\Omega_{\text{b}} > 10^2$ – 10^3 , which is consistent with the observed value $\Omega_{\text{b}} \sim 0.003$. However, the maximal possible value $1/\kappa \sim R_{\text{H}}$ (R_{H} is the Hubble radius), which gives $\Omega_{\text{DM}}/\Omega_{\text{b}} \sim 10^5$ cannot be excluded. To avoid any misunderstanding, we stress that the topological nature of the bias causes the fractal distribution to reach equilibrium and be consistent with the homogeneity of the metric and the observed CMB fluctuations $\Delta T/T$ (e.g. see Kirillov 2006). The topological nature means that the same bias appears in all interactions. If the bias did not modify the electromagnetic field, then the fractal distribution of baryons would be in severe conflict with observations and surely would have to be rejected as it does take place in the standard models (e.g. the fractal distribution produces too strong fluctuations $\Delta T/T \sim \Delta \rho_{\text{b}}/\rho_{\text{b}} \sim \mu/\kappa$). The topological nature of the bias however creates the fact that the Coulomb force and all Green functions are also modified at galaxy scales (Kirillov & Turaev 2002; Kirillov 2006) which reduces $\Delta T/T$ to the observed value $\Delta \rho_{\text{total}}/\rho_{\text{total}} \sim \kappa/\mu \sim 10^{-5}$ [e.g. for sufficiently remote objects $r \gg \langle R_0 \rangle$, the apparent luminosity has to behave as $\ell \sim L/r^{D-1}$ instead of $1/r^2$, which gives the number of objects brighter than ℓ the somewhat higher (with respect to $D = 3$) value $N(\ell) \approx \nu r^D(\ell) \approx 1/\ell^{D/(D-1)}$]. Thus the topological bias and the observational definition of $1/\kappa$ requires the careful and thorough revision of the standard model and all basic formulas.

In conclusion, we point out that bias (6) predicts the existence of specific oscillations in the distribution of DM with the characteristic wavelength $\sim M_{\text{g}}^{1/D} \sim \sqrt{L}$. When the observational data allow, this can be used to verify the theory and, thus, to make a more definite conclusion on the nature of DM.

ACKNOWLEDGMENT

We acknowledge P. Salucci's valuable comments and the advice of referees which helped us to essentially improve the presentation of this work. This research was supported in part by the Center for Advanced Studies in the Ben-Gurion University of the Negev.

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