

CORRECTIONS TO THE NEWTON AND COULOMB POTENTIALS CAUSED BY EFFECTS OF SPACETIME FOAM

A.A. Kirillov^{1†} and D. Turaev^{2‡}

† Institute for Applied Mathematics and Cybernetics, 10 Ulyanova St., Nizhny Novgorod 603005, Russia

‡ Weierstrass Institute for Applied Analysis and Stochastisity, Mohrenstrasse 39, D-10117, Berlin, Germany

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We use the modified field theory (MOFT), previously suggested by one of the authors [1], to explore possible observational effects of the spacetime foam. It is shown that, as was expected, the spacetime foam can provide quantum Bose fields with a cutoff at very small scales if the energy of zero-point field fluctuations is taken into account. It is also shown that MOFT changes the behaviour of massless fields at very large scales (in the classical region). In particular, we show that at $r \gg r_0$ the Coulomb and Newton forces acquire the behaviour $\sim 1/r$ instead of $1/r^2$.

О поправках к кулоновскому и ньютоновскому потенциалам, вызванных эффектами пространственно-временной пены

А.А. Кириллов, Д. Тураев

Рассматривается модифицированная теория поля (МОФТ), предложенная ранее одним из авторов [1], для исследования возможных наблюдаемых эффектов пространственно-временной пены. Показано, что при учете энергии нулевых колебаний полей наличие пены может приводить к появлению масштаба ультрафиолетового обрезания для бозонных полей. Кроме того, показано, что модификация теории поля приводит к изменению поведения безмассовых полей на очень больших масштабах (в классической области). В частности, показано, что при $r \gg r_0$ кулоновская и ньютоновская силы приобретают поведение $\sim 1/r$ вместо $1/r^2$.

It is commonly believed that quantum gravity effects (spacetime foam) should provide a cutoff for quantum field theory. A Modified field theory (MOFT), which suggests a way to account for the spacetime foam effects, was put forward by one of us in Ref. [1]. MOFT extends standard quantum fields theory to the case when the number of Bose fields becomes variable and introduces a new operator $N(k)$ which is the density of number of fields in the momentum space. Standard QFT represents a particular sector of MOFT in which $N(k) = 1$. In this letter, we show that this operator causes a modification of the Green functions $\bar{D}(k) = D(k)N(k)$ in all internal lines of the diagram technique. At very small scales $k \ll k^*$ it indeed can provide a cutoff if we take into account the energy of zero-point field fluctuations. On the other hand, for massless fields this operator causes essential modification at large scales $k \ll \mu$ (μ is the Fermi energy, see, e.g., Ref. [1]).

We first note that, from a phenomenological viewpoint, the operator $N(k)$ reflects the topology of physical space. In the modern Universe, topology changes

are strongly suppressed. There exist severe experimental restrictions which come from oscillation experiments (see, e.g., Ref. [2] and references therein). This means that now a superselection rule acts, and the operator $N(k)$ represents a constant of motion and may be considered as an ordinary function. However, in the early Universe, during the quantum stage, topology changes took place, and the properties of the function $N(k)$ were formed then.

Consider, as an example, the modification of quantum electromagnetic field. The electromagnetic interaction operator is described by the term $V = e \int j^\mu A_\mu d^3x$ (where e is the electron charge and j^μ is the current density of sources). In MOFT the number of fields is variable, we have A_μ^a ($a = 0, 1, \dots$), and one can say that there exist photons of different “sorts”. However, these sorts are indistinguishable (fields are supposed to obey the identity principle, see, e.g., Ref. [1]), and therefore the interaction term should include additional summing over the sorts of photons $V = e \sum_a \int j^\mu A_\mu^a dx$. We note that the same summing appears in the total Hamiltonian describing free photons.

The Green function for photons in the coordinate

¹e-mail: kirillov@unn.ac.ru

²e-mail: turaev@wias-berlin.de

representation has the form (see, e.g., Ref. [3])

$$D_{\mu\nu}(x-x') = i \left\langle T \left(A_\mu(x) A_\nu(x') \right) \right\rangle, \quad (1)$$

where averaging is taken over the vacuum state and the symbol T denotes chronological ordering. A Fourier transform of this function in the Feynman gauge is expressed in terms of the scalar function $D(k)$:

$$D_{\mu\nu}(k) = g_{\mu\nu} D(k). \quad (2)$$

In standard quantum theory, in the case of free fields, $D(k) = 4\pi/k^2 = 4\pi/(\omega^2 - \mathbf{k}^2)$ (here $k_\mu = (\omega, \mathbf{k})$). While considering photons of a particular ‘‘sort’’, the same is also valid in MOFT: $D^a(k) = 4\pi/k^2$. However, it is obvious that in perturbation theory all diagrams will include additional summing over the sorts of photons. Such a summing can be carried out explicitly, and therefore it is sufficient to consider a modified Green function:

$$\tilde{D}(k) = \sum_a D^a(k) = \frac{4\pi N(k)}{k^2}. \quad (3)$$

A nontrivial fact is here that MOFT admits a nontrivial ground state in which the number of fields $N(k)$ contains a dependence on the wave vector k [1].

From a formal point of view, the expression (3) breaks the Lorentz symmetry. This, however, does not mean that the extended theory is not Lorentz-invariant. The symmetry breaks due to a particular choice of the field ground state Φ_0 , which was shown to possess finite energy and particle number densities [1]. Such particles are dark and cannot be directly observable, but should contribute to the dark matter. Therefore, the state Φ_0 distinguishes a particular preferred reference frame. The ground state Φ_0 represents an eigenstate of the total momentum operator $\hat{P}_\mu \Phi_0 = p_\mu \Phi_0$. In the preferred reference frame, the eigenvalue p_μ has the components $p_\mu = (E_0, 0, 0, 0)$. Thus in an arbitrary reference frame the function $N(k)$ can be presented in the form $N(k) = N(s)$ where $s = k_\mu p^\mu / E_0$.

Since in all internal lines of the diagram technique the function $\tilde{D}(k)$ is multiplied by e^2 , we can interpret the modification (3) as if all point charged particles acquire the dispersion $e^2(k) = e^2 N(k)$, i.e., in MOFT point sources turn out to be distributed in space.

As was shown in Ref. [1], the field ground state Φ_0 can be characterized by occupation numbers of the type

$$N_{k,n} = \theta\left(\mu - [n\omega + \Delta(k)]\right), \quad (4)$$

where $N_{k,n}$ is the number of field modes in the quantum state (k, n) (n is the number of photons in a given mode), $\theta(x)$ is the Heaviside step function, μ is the chemical potential, and we have added the term $\Delta(k)$ which is the minimal energy of field modes (vacuum spectral energy density). We note that the current state of field theory does not allow one to fix the form of

$\Delta(k)$, and in what follows we shall not specify it. The standard definition $\Delta(k) = \frac{1}{2}\omega$ results in an infinite energy density in QFT and requires renormalization, while in our extended quantum field theory (EQFT) this choice produces a too small value for the cutoff (see below). However, we may expect that $\Delta(k) \sim \omega > 0$ is an increasing function. Thus, for the mode spectral density we get

$$N_k = \sum_{n=0}^{\infty} \theta(\mu_k - n\omega) = \left[1 + \frac{\mu_k}{\omega}\right], \quad (5)$$

where $[x]$ denotes the integer part of the number x and $\mu_k = \mu - \Delta(k)$. Eq. (5) shows, in particular, that $N_k = 0$ when $\mu_k < 0$, and therefore there appears a cutoff k^* whose value is a solution of the equation $\mu - \Delta(k^*) = 0$. The standard picture of the electromagnetic field is only valid in the wavenumber range ($\omega = \omega_k = \sqrt{\mathbf{k}^2}$)

$$\omega_k > \mu_k > 0 \quad (6)$$

where we have $N_k = 1$. In what follows, for the sake of simplicity, we set $\mu_k = \mu$ (thereby neglecting the existence of the field energy of zero-point fluctuations and of the respective cutoff). Then, (5) reads $N_k = 1 + [\mu/\omega]$, and in the range $\omega < \mu$ we find a correction to the standard Green function:

$$\tilde{D}(k) - D(k) = \frac{4\pi}{k^2} \left[\frac{\mu}{\omega} \right]. \quad (7)$$

We note that the above consideration also remains valid in the case of the linearized gravitational field $h_{\mu\nu}$ (gravitons) with the replacement of Eq. (2) with $D_{\mu\nu,\alpha\beta}(k) = \frac{1}{2}(g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha})D(k)$ and the same function $D(k)$ as in (2).

Consider now corrections to the Coulomb and Newton laws. Since the number of fields is variable, the interaction energy between two particles V must contain a sum over all sorts of photons or gravitons: $V = \sum_a V^a$. Consider two pointlike particles at rest. Then the Fourier transform of the correction $\delta V(k)$ to the standard Coulomb potential energy

$$V(k) = 4\pi e^2 Z_1 Z_2 / |\mathbf{k}|^2$$

takes the form

$$\delta V(\mathbf{k}) = \frac{4\pi e^2 Z_1 Z_2}{|\mathbf{k}|^2} \left[\frac{\mu}{|\mathbf{k}|} \right], \quad (8)$$

where $Z_{1,2}$ are the particle electric charge values (in the case of gravity one should use the obvious replacement $e^2 Z_1 Z_2 \rightarrow -Gm_1 m_2$). In the coordinate representation, this potential is given by the integral

$$\delta V(r) = \frac{1}{2\pi^2} \int_0^\infty (\delta V(\omega) \omega^3) \frac{\sin(\omega r)}{\omega r} \frac{d\omega}{\omega}. \quad (9)$$

Since $\delta V(\omega)$ vanishes for $\omega > \mu$, the upper limit of this integral is $\omega = \mu$. At the low limit $\omega = 0$ this integral is divergent. However, we note that the interaction between particles can exist only on scales smaller than the

horizon size ℓ_h . Thus we must take as the lowest limit $\omega \sim 1/\ell_h \sim H$ where H is the Hubble constant. This integral can be presented in the form

$$\delta V(r) = \sigma \sum_{n=1}^{N^*} \int_H^{\mu/n} \frac{\sin(\omega r)}{\omega r} d\omega, \quad (10)$$

where $\sigma = 2e^2 Z_1 Z_2 / \pi$ and $N^* = \mu/H$. In the range $\mu r \ll 1$ this correction produces a constant shift (which gives a finite contribution to the electromagnetic rest mass of the particle $\delta m = \delta V$)

$$\delta V \sim \sigma \mu \sum_{n=1}^{N^*} \frac{1}{n} \sim \sigma \mu \ln \left(\frac{\mu}{H} \right). \quad (11)$$

At the opposite asymptotic $\mu r \gg 1$ (but $Hr \ll 1$) we find the estimate

$$\delta V \sim -\sigma \mu \ln(Hr). \quad (12)$$

The expression (12) shows essential deviations from the Newton and Coulomb laws at scales $r > r_0 \sim 1/\mu$. In particular, at these scales the Newton and Coulomb forces acquire the behaviour $1/r$ (instead of $1/r^2$). We note that the value r_0 can be very large, and it is therefore impossible to carry out a direct observation of the correction to the Coulomb potential (at macroscopic scales the number of positive and negative charges is equal, and the potential vanishes long before reaching the scale r_0). However, for gravity the situation is different since the gravitational potential is accumulated, and the correction (12) must leave an imprint in astronomical observations.

And indeed, there exists an indication that such a behaviour really takes place. As is well known, the observations show that a leading contribution to the distribution of matter is given by the so-called dark matter which should have an exotic non-baryonic form and is not directly observable (see, e.g., [4]). There are several observations which provide evidence for dark matter. One is connected with measurements of the rotational velocity of galaxies as a function of the radial distance from the centre, the so-called rotation curve (see, e.g., [4, 5]). According to the standard Newton dynamics, the rotation curve of a disk with an internal mass distribution that follows from the observed brightness law must show a Keplerian $r^{-1/2}$ behaviour at large radii. However, measurements [5] show that $v(r) = v_m$ remains constant, which implies that the total mass contained within a radius r , $M(r)$, varies with r . Indeed, according to the standard Newton law, the acceleration of a body in a circular orbit of radius r is $a = GM(r)/r^2 = v^2(r)/r$, which gives $M(r) = v_m^2 G^{-1} r$. This can be interpreted as if the mass per unit luminosity M/L increased with radius, and therefore a large fraction of the total mass of a galaxy is in the form of a non-luminous, dark component located at large radii. However, if we take into

account the correction (12), we find that, for $r > r_0$, $v^2(r) = v_m^2 \sim 2\mu GM/\pi$ is consistent with the light distribution ($M/L \sim \text{const}$). It is not yet clear which fraction of the dark matter can be explained by the correction (12) to the Newton potential (we recall that the ground state Φ_0 itself unavoidably predicts the existence of dark matter [1]). However, this allows one to give a tentative rough estimate of the characteristic scale $r_0 > 1$ kpc, and so the parameter μ is really small. Obtaining a more precise estimate requires a further and more thorough confrontation with observations.

We note that the idea of modifying gravity at large scales is not new, see, e.g., the criticism of different approaches and a list of references in Ref. [6]. Some approaches use potentials close to (12) as an empirical requirement to the modification of the Newton law (or, equivalently, introduction of an additional force) — see, e.g., [7] — and have no fundamental theoretical background. On the contrary, in the extended quantum field theory the correction (12) is an inevitable consequence of the massless nature of the gravitational field.

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